APPLICATION SIZING GUIDE

Servomotor Selection

Introduction
Kollmorgen offers two fundamental motor technologies for servo systems. They are DC brush type and Brushless DC (permanent magnet synchronous). Both have unique features and benefits. In some cases, based on the performance requirements of the application, only one particular technology of product will be appropriate. Other times, motor options or the features of the servo drive amplifier may dictate the type of motor chosen. In many applications however, either motor type might meet the need. The decision of which to choose could then be made based on cost or maintenance issues.

DC brush type motor and amplifier systems are generally lower cost in low power ratings due to their simpler design. DC technology provides reliable performance for applications with more limited duty operation. DC motors remain the technology of choice for applications requiring precision low speed performance.

For most applications however, brushless servos have emerged as the preferred technology. This is based on the performance benefits including: minimized maintenance, lower rotor inertia allowing faster accelerating machinery, higher speed operation, efficient heat dissipation and operation in special environments. Brushless motors require position feedback for electronic commutation, thus the amplifiers for these motors are more complicated than DC types. This typically results in more cost for the same power rating as compared with DC.

Motor Sizing
Basic Considerations:

- Thrust Loads
- Friction Losses
- Acceleration Requirements
- Root Mean Square (RMS) Torque Evaluation
- System Component Requirements (Motor, Amplifier, Power Supply)

Motor sizing begins with the evaluation of the load relative to torque and speed requirements. In the simplest of cases, where the load is constant, the product can be selected easily from the catalog. More often however, product selection is based on load changes relating to among other things, acceleration torque as a function of load plus motor inertia.

Evaluation of the load requirements can be divided into two major categories:

1) Forces required to accommodate the desired task, including:
   a) Cutting or thrust forces
   b) Counterbalancing forces (vertical axes)
   c) Acceleration forces taking into account the total system inertia

2) Forces to overcome frictional losses including:
   a) Bearing surfaces losses
   b) Screw or rack and pinion inefficiencies
   c) Reducer inefficiencies
   d) Static and dynamic motor losses
The calculated torques are summed when acting concurrently and then applied together based on the overall machine duty cycle. A root-mean-square (RMS) calculation is used to determine the effective torque required by the motor to be compared with the continuous torque rating of a motor. The process of choosing a motor most often requires iterative calculations since the motor inertia is a factor in determining the acceleration torque and RMS requirements. In addition to the RMS requirement, the motor/drive combination chosen must be able to provide peak torque for the required duration.

Consideration should be made for the motor thermal time constant if the on time at a peak torque value is for any substantial period. Typical complementary amplifiers, having a two times peak current rating for 2 to 8 seconds, do not present a problem even for small motors having a short thermal time constant. Problems may occur however if an amplifier is "oversized" for the motor meaning that the drive continuous current is greater than the motor continuous rating. In these cases, other evaluation methods can be used taking into account this motor time constant.

Once a motor is chosen that meets the torque requirement of the application, the minimum speed winding should be chosen to satisfy the required application speed. A penalty is paid in terms of the increased amplifier size and cost (increased current) required for a motor having higher speed capability (having lower $K_T = \text{torque/amp}$).
Inertia Calculations

**Obtaining the Inertia of a Solid Cylinder:**

\[ J = \frac{Mr^2}{2} = \frac{W \times r^2}{g} \times \frac{r^2 \times L \times \rho}{C_2} \]

Use to calculate inertia of pulleys, lead screws, shafts, couplings, etc.

Where:
- \( J \) = Load Inertia
- \( M \) = Mass
- \( W \) = Weight of material
- \( r \) = Radius of cylinder
- \( L \) = Length of cylinder
- \( \rho \) = Specific gravity of cylinder material
- \( g \) = Gravitational acceleration
- \( C_2 \) = Unit conversion factor

**UNITS**

\[
\begin{array}{cccccc}
\text{J (Inertia)} & \text{r} & \text{L} & \rho_{(\text{alum.})} & \rho_{(\text{steel})} & C_2 \\
\text{lb-ft-s}^2 & \text{in} & \text{in} & 0.0955 \text{ lb/in}^3 & 0.284 \text{ lb/in}^3 & 2952 \\
\text{lb-in-s}^2 & \text{in} & \text{in} & 0.0955 \text{ lb/in}^3 & 0.284 \text{ lb/in}^3 & 246.0 \\
\text{oz-in-s}^2 & \text{in} & \text{in} & 1.53 \text{ oz/in}^3 & 4.54 \text{ oz/in}^3 & 246.0 \\
\text{kg-cm}^2 & \text{cm} & \text{cm} & 0.00265 \text{ kg/cm}^2 & 0.00788 \text{ kg/cm}^2 & 623.9 \\
\text{kg-m}^2 & \text{cm} & \text{cm} & 0.00265 \text{ kg/cm}^2 & 0.00788 \text{ kg/cm}^2 & 6366 \\
\end{array}
\]

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**Obtaining the Inertia of a Hollow Cylinder:**

\[ J = \frac{Mr^2}{2} \]

Use to inertia of belts, weights on conveyors, rings, wound material, etc.)

Where:
- \( J \) = Load inertia
- \( M \) = Mass
- \( r \) = Radius of weight (or mass)
- \( W \) = Weight of material
- \( g \) = Gravitational acceleration

**UNITS**

\[
\begin{array}{cccccc}
\text{J (Inertia)} & \text{r} & \text{W} & \text{g} \\
\text{lb-ft-s}^2 & \text{ft} & \text{lb} & 32.2 \text{ ft/s}^2 \\
\text{lb-in-s}^2 & \text{in} & \text{lb} & 386 \text{ in/s}^2 \\
\text{oz-in-s}^2 & \text{in} & \text{oz} & 386 \text{ in/s}^2 \\
\text{kg-cm}^2 & \text{cm} & \text{kg} & \text{---} \\
\text{kg-m}^2 & \text{m} & \text{kg} & \text{---} \\
\end{array}
\]
**Obtaining the Inertia of a Slide:**

\[ J_s = \frac{WL^2}{C_3\eta_s} \]

Where:  
- \( J_s \) = Inertia of slide and part  
- \( W \) = Weight of slide (\( W_s \)) plus weight of part (\( W_p \))  
- \( L \) = Lead (displacement / rev.) of screw  
- \( C_3 \) = Unit conversion factor  
- \( \eta_s \) = Efficiency of screw (typical efficiencies: Ballscrew - .85 to .95; Acme screw - .40 to .60)

**Note:** The efficiency term is used in the inertia calculation to take into account losses when calculating acceleration torque

### Units

<table>
<thead>
<tr>
<th>J (Inertia)</th>
<th>W</th>
<th>L</th>
<th>C_3</th>
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</thead>
<tbody>
<tr>
<td>lb-ft-s^2</td>
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<td>cm/rev</td>
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</tr>
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</table>
**Torque Calculations**

**Obtaining Load Torque:**

\[ T = F \times r \]

Where:  
- \( T \) = Load Torque (Typical units: lb-ft, lb-in, oz-in, N-m)  
- \( F \) = Perpendicular force to rotate a shaft  
- \( r \) = Radius to apply the force \( F \)

**Obtaining the Torque for a Slide:**

\[ T = \frac{(F + \mu W) \times L}{C_1 \eta_s} \]

Where:  
- \( T \) = Load torque  
- \( F \) = Cutting force  
- \( W \) = Weight of slide (\( W_s \)) plus weight of part (\( W_p \))  
- \( \mu \) = Coefficient of friction  
- \( L \) = Lead of screw (displacement / revolution)  
- \( C_1 \) = Unit conversion factor  
- \( \eta_s \) = Efficiency of the screw (typical efficiencies: Ballscrew - .85 to .95; Acme screw - .40 to .60)

**UNITS**

<table>
<thead>
<tr>
<th>T (Torque)</th>
<th>F</th>
<th>W</th>
<th>L</th>
<th>C_1</th>
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<tbody>
<tr>
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<td>lbs</td>
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<td>lbs</td>
<td>in/rev</td>
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<td>oz</td>
<td>in/rev</td>
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<td>kg-cm</td>
<td>kg</td>
<td>kg</td>
<td>cm/rev</td>
<td>2\pi</td>
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</table>
**Constant Torque Acceleration:**

\[ T(torque) = J(inertia) \alpha(acceleration) + T_f(friction \ torque) \]

or

\[ T(torque) = \frac{J(inertia) \cdot N(RPM)}{C_4 \cdot t(seconds)} + T_f(friction \ torque) \]

Where:  
\( C_4 = \text{Unit conversion factor} \)  
\( t = \text{Acceleration time in seconds} \)

<table>
<thead>
<tr>
<th>UNITS</th>
<th>T(torque)</th>
<th>T_f (friction torque)</th>
<th>J (inertia)</th>
<th>C_4</th>
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<td>Kg-cm</td>
<td>Kg-cm</td>
<td>Kg-cm-s^2</td>
<td>9.55</td>
<td></td>
</tr>
</tbody>
</table>

**RMS Torque:**

\[ T_{RMS} = \sqrt{\frac{T_1^2 t_1 + T_2^2 t_2 + T_n^2 t_n}{t_1 + t_2 + t_n}} \]

Where:  
\( T_1, T_2, T_n = \text{Torques 1-n} \)  
\( t_1, t_2, t_n = \text{Times 1-n} \)
### Power Equations

\[
\text{Power} = \frac{N(RPM) \times T}{C_1}
\]
\[
\text{Power (HP)} = \frac{\text{Watts}}{746}
\]

Where:  
- \(T\) = Torque  
- \(C_1\) = Unit conversion factor

#### UNITS

<table>
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<tr>
<th>Power</th>
<th>T</th>
<th>(C_1)</th>
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<td>HP</td>
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<td>Watts</td>
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<tr>
<td>Watts</td>
<td>kg-cm</td>
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### Other Considerations

#### Option Requirements

Servomotors often require modifications necessary to meet the complex requirements of a machine. Standard options for the various models of servomotors include but are not limited to: gearheads; fail-safe brakes; environmental sealing; feedback devices including tachometers, resolvers, and encoders; and connectors. Special options include modifications to shaft and mounting surfaces, special windings and fan cooling provisions to mention a few.

#### Ambient Temperature

Operation of motors in ambients other than that for which the motor is rated may require special consideration. Operation in ambients less than the rated temperature will result in a greater continuous torque rating but excessively cold ambients may require a different bearing lubrication. Operation in ambients above the rated temperature results in a derating of the motor continuous torque. The derating will be a factor of the motor design and its ultimate temperature rating.

#### Gear Reducers

Gearmotors offer the following advantages to a servo application:

- The ability to operate the motor over its optimum speed range
- Minimize motor size by multiplying torque
- Minimize reflected inertia for maximum acceleration
- Provide maximum torsional stiffness

The following relationships apply:

- \(\text{Output Speed} = \frac{\text{Input Speed}}{\text{Gear Ratio}}\)

Where: \(\text{Gear Ratio} = \frac{\text{Turns of the motor}}{\text{Turns of the load}}\)
Output Torque = Input Torque * Gear Ratio * Gearbox Efficiency

Where: Gearbox Efficiency = \( \frac{\% \text{ of Efficiency}}{100} \)

\[ J_c(\text{reflected}) = \frac{J_L}{\text{Gear Ratio}^2 \eta_S} + J_{\text{Gearbox}} \]

Where: \( J_L \) = Load inertia
\( J_{\text{Gearbox}} \) = Gearbox inertia
\( \eta_S \) = Efficiency of gearbox

Note: The efficiency term in the denominator is used when making acceleration calculations to account for the additional input torque required to overcome gearbox losses.

Optimum Gear Ratio (n) = \[ n = \sqrt{\frac{J_L + J_{\text{Gearbox}}}{J_M}} \]

Where: \( J_M \) = Motor inertia

There are two basic approaches to sizing gearmotors for servo applications. The simplest approach can be used if the load is basically constant and acceleration or deceleration rates are not a consideration. This requires selection based on known load torques and speeds. For this example however, it may still be necessary to calculate the reflected inertia of the load for amplifier compensation purposes. It is desirable to keep the reflected inertia ≤ 5xJ_m (motor inertia). This may affect the selection of the motor series.

Generally however, with servo applications, peak and continuous torque requirements are determined by an analysis of the desired motion profiles and duty cycle. This is done by the following procedure.

1. Reflect load inertias to the motor by a ratio (n) as close to optimum as possible. The optimum gear ratio will yield a reflected inertia equal to the motor inertia. With gearbox input speeds limited to 3000 to 4000 RPM, depending on model, this is not always possible. It is desirable to keep the reflected inertia under 5 times the motor inertia.

2. Calculate acceleration torques, along with thrust or friction torques, neglecting motor inertia at this time. Determine the RMS torque. An initial motor/amplifier selection can now be made.

3. Calculate the ratio of load to motor inertia (n) based on the motor chosen. Include the gearbox inertia with the load inertia.

\[ n = \frac{J_c(\text{reflected}) + J_{\text{gearbox}}}{J_M} \]

Should (n) be greater than 5, consider a motor with more inertia.

4. Recalculate acceleration and RMS torques based on the system selected. Include now the motor inertia as part of the total system inertia to calculate the acceleration torque required.

5. Based on the true acceleration and RMS torque values determined in the previous step, check the motor torque rating. A recalculation may be necessary using a motor with a higher torque rating.
Direct Drive (Frameless) Motor Considerations
A direct drive motor is a servo actuator which can be directly attached to the load it drives, thereby eliminating backlash and increasing servo stiffness. Direct drive motors are well suited to applications where it is desired to minimize size, weight, response time and required power input, while maximizing positional accuracy. These frameless motors require particular consideration with respect to torque sizing and installation. The continuous torque rating of a frameless motor will be based on the heat sinking ability of structure in which it is mounted.

Considerations for installing brushed verses brushless direct drive motors are somewhat different. Brushed motors are available with either rare earth or alnico type magnets. Motors having alnico magnets in the stator require special handling. Alnico magnet designs require a ‘keeper ring’ which provides a flux path prior to installation of the wound armature. Removal of the keeper ring before insertion of the armature will result in demagnetization of an alnico stator. In addition, alnico magnet stators require a minimum of 1/2 inch of non-magnetic material outside the stator so as not to reduce the flux density through the armature.

Disc Armature Motors
Disc armature motors are a unique design of DC motor having an ironless flat disc armature rather than the conventional iron lamination based cylindrical armature. As a result, the disc armature motor has low inertia, and due to its low inductance, has low RFI and EMI characteristics. The armature design results in a very low profile package. This motor finds applications in both open loop applications and servo systems with the tachometer option. A wide variety of other options including gearheads, brakes, feedback devices, and complementary amplifiers are available
Amplifier Selection

Introduction
A servo amplifier is needed to control the power delivered to the motor and ultimately to control the system. With a DC motor amplifier, power is provided to a self-commutating DC motor. In a brushless motor amplifier, additional circuitry is required to electronically commutate power to the motor windings so rotation can occur. Servo amplifiers compare actual motor position or velocity feedback signals with the command input signals and adjust the power output level to the motor.

Feedback and command signal comparison provides closed loop operation for precise motion control. Torque, which is proportional to current, can be monitored through current sensors inside the amplifier (current loop mode of operation). Motor velocity can be monitored using tachometer, resolver, Hall device, or encoder feedback (velocity loop of operation). Commutation circuitry in a brushless motor servo amplifier, switches power to the different windings in a way analogous to a conventional motor's brushes and commutator. To do this it relies on a position feedback device connected to the motor shaft.

The servo amplifier is an essential component for system speed and torque control. Additional control features such as a position loop may be added to the system or integrated into the amplifier.

Amplifier Sizing
Amplifier sizing is initially an issue of equating the power requirement of an amplifier to that of a motor. Servo amplifiers typically have both continuous and peak current ratings along with a maximum voltage rating. The peak current rating is available for acceleration/deceleration requirements or for transient loads, and is typically available for two or more seconds. Voltage and current ratings of an amplifier need to be matched with a motor's winding constants to yield the desired performance. Considerations such as winding losses and viscous damping losses need to be considered. A commonly adopted approach to minimize sizing errors has been the development of system performance curves. These curves depict the resulting output performance of a motor and amplifier combination. This greatly aids the servo system designer.

Mode of Operation
Considerations for the mode of operation, feedback device type, I/O and other product features must be made. Typical amplifier modes of operation include: torque(current) mode, velocity (servo) mode, speed (limited speed range) mode, and position mode. The desired mode of operation will often dictate the amplifier model(s) that can be chosen. The amplifier type and model in turn may dictate the feedback device requirement for the motor.

Digital or Analog
An important consideration in the selection of brushless motor amplifiers is the decision for analog or digital technology. Analog technology has been a standard for brush and brushless motor amplifiers. With analog amplifiers a voltage reference command results in speed or torque, depending on the mode of operation, proportional to the command. As well, compensation (tuning) of the amplifier is done by analog circuitry requiring potentiometer/switch and, or compensation "comp" board changes for different system configurations (mode of operation and motor & load variations). Digital electronics on the other hand, enhance drive features and minimize set-up concerns. The command reference for digital drives can be done by either analog or digital inputs. Digital input options are typically in a pulse format or through an RS 232 or 485 interface. Typically, better regulation of the system can be obtained by the digital command format. Digital drives also have the feature of tuning through a digital interface. This greatly benefits the issue of load compensation relative to inertia, friction and compliant factors which are often difficult and time consuming to simulate under lab conditions. Another benefit of digital technology is the diagnostics enhancements it offers. Monitoring of internal functions of the drive such as following error or current can be easily done. Fault diagnostics are typically more detailed than with analog drives and fault histories are maintained through power loss.
**Six-step or Sinusoidal Commutation**

Another consideration in amplifier selection, as it relates to brushless motor systems, is the type of electronic commutation. Two basic types of commutation are available, six step sequencing and sinusoidal. Six step commutation is the simplest requiring Hall Effect feedback devices mounted in the motor. This technique is lower cost to implement yet produces more system ripple than the sinusoidal commutation approach. For low speed velocity control, these systems need an additional high resolution feedback device mounted in the motor. Sinusoidal commutation continuously varies the current to the motor phases providing more precision motion control. These amplifiers require a high resolution resolver or encoder to be mounted in the brushless servomotor.
Power Supply Selection

Introduction
The power supply in a servo system fundamentally delivers DC power to a servo amplifier. This is done by the rectification and filtering of AC power delivered from a 1∅ or 3∅ main. Depending on the specific servo amplifier chosen, the power supply may be internal to the amplifier, for which no selection process is required, or external, thus requiring selection decisions. In addition to the basic power sizing requirement, typical DC supplies for servo amplifiers include shunt regeneration circuitry which dissipates regenerative energy from the load when the motor is decelerated rapidly. The following information gives general guidelines for power supply and regeneration selection.

Power Supply Sizing
The power output rating of a power supply must exceed or equal the combined average power of all servo drives operating simultaneously. The average power of an individual servo amplifier is based on the power calculation of RMS torque and speed. Taking into account motor and drive losses, as a general rule, for permanent magnet servos (DC brush or brushless), 1 kW of input power is needed for every 750 watts (approximately 1 HP) of output power.

\[
\text{Power(Watts)} = \frac{N \times (\text{RPM}) \times T}{C_1}
\]

<table>
<thead>
<tr>
<th>UNITS</th>
<th>T (Torque)</th>
<th>C_1</th>
</tr>
</thead>
<tbody>
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</tr>
<tr>
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</tr>
<tr>
<td>N-m</td>
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</tbody>
</table>

In addition, power supplies often supply the logic power for the servo amplifier. The voltage ratings and ampacity of this power supply must also be appropriate for the amplifiers being supplied.

Regeneration Component Sizing
Regeneration is the process whereby kinetic energy of a motor and load is converted into electrical energy during deceleration by a four quadrant servo controller. Regeneration also occurs if motor rotation is opposite to that of the commanded torque as in an unwind web application. Dissipation of this electrical energy can be done by one of two methods. These are line regeneration and shunt (resistive) regeneration.

Line regeneration is a more costly approach which involves the conversion of DC power into AC power, synchronous with the AC source. In battery operated systems, regeneration into the battery (recharging) will occur without the need for additional electronics.

Shunt regeneration is accomplished by the shunting (short circuiting) of the DC bus through a power resistor. This technique is more common and can handle higher transient loads than line regeneration. For decelerating loads, the amount of regeneration required is a function of the sum of simultaneously decelerating inertial loads where multiple amplifiers are being operated from a common supply. The loads need to be defined in terms of the system inertia (load plus motor inertia), maximum speed, and the deceleration time from maximum speed. These parameters define the kinetic energy that must be either stored or dissipated. The power supply has some capacity to store some of this energy, and some energy will be dissipated in the motor windings and amplifier circuitry. Any remaining energy will need to be dissipated in an appropriately sized regeneration system.

Transformer Selection
Transformers serve two basic purposes in a servo system: voltage matching and isolation from the line. Voltage matching is fundamental and required for many systems where standard AC line voltages like 115, 1∅ and 230 or 208, 3∅ are not appropriate. Line isolation is required for some products or can benefit other applications by the reduction of PWM noise for sensitive environments.
Application Example: Slide Drive

General Information:
This system represents a typical screw driven slide mechanism incorporated into a wide variety of machine tool and automation equipment.

Customer Specifications:
Axis: X (horizontal)
Slide:
- Slide weight: 750 lbs
- Max. part weight: 500 lbs
- Coef. of friction: 0.1
- Max. thrust force: 2500 lbs
- Duty Cycle: 100%
- Max. linear speed: 500 IPM
- Max. cutting speed: 200 IPM

Lead Screw:
- Lead: 0.5 in/rev
- Diameter: 1.5 in
- Length: 48 in
- Efficiency: 90%

Speed Reducer:
Optional, belt or gearhead ratio may be suggested to optimize motor selection.

Max. acceleration rate: 200 inches/sec^2

Incremental encoder feedback resolution minimum: 4000 counts/rev

Inertia Calculations:

Inertia of a slide

\[ J_{\text{Slide}} = \frac{W \times L^2}{C \eta_s} \]

Units:  
- \( J \) = Inertia in lb-ft-s^2  
- \( W \) = Weight in lbs  
- \( L \) = Lead in in/rev  
- \( C \) = Unit conversion factor = 182900  
- \( \eta_s \) = Efficiency of screw in %/100

\[ J_{\text{Slide}} = \frac{(750 + 500) \times 0.5^2}{182900 \times 0.9} = 0.00190 \text{ lb} \cdot \text{ft} \cdot \text{s}^2 \]

Inertia of a solid cylinder (ballscrew)

\[ J_{\text{Screw}} = \frac{r^4 \times L \times \rho}{C} \]

Units:  
- \( J \) = Screw inertia in lb-ft-s^2  
- \( r \) = Radius of screw in inches  
- \( L \) = Length of screw in inches  
- \( \rho \) = Specific gravity of steel in lb/in^3 = 0.284  
- \( C \) = Unit conversion factor = 2952

\[ J_{\text{Screw}} = \frac{\left(\frac{1.5}{2}\right)^4 \times 48 \times 0.284}{2952} = 0.00162 \text{ lb} \cdot \text{ft} \cdot \text{s}^2 \]

Load Calculations:

Obtaining the torque requirement for a slide

\[ T (\text{total torque}) = T_{\text{Cut}} (\text{cutting torque}) + T_f (\text{friction torque}) \]

\[ T = \frac{F \times L}{C \eta_s} + \frac{\mu W \times L}{C \eta_s} \]

Units:  
- \( T \) = Torque in lb-ft  
- \( F \) = Cutting force in lbs  
- \( \mu \) = Coefficient of friction  
- \( W \) = Weight of part plus slide in lbs  
- \( L \) = Lead of screw in in/rev  
- \( C \) = Unit conversion factor = 75.4  
- \( \eta_s \) = Efficiency of screw in %/100
Application Example: Slide Drive continued

Load Calculations (contd.):

\[ T = \frac{2500 \times 0.5}{75.4(0.90)} + \frac{0.1(500 + 750) \times 0.5}{75.4(0.90)} \]

\[ T = 18.4 + 0.9 = 19.3 \text{ lb } \cdot \text{ft} \]

Speed Calculations:

\[ N(\text{RPM}) = \frac{\text{Linear Speed (IPM)}}{\text{Lead (in/rev)}} \]

Maximum speed: \[ \text{RPM} = \frac{500}{0.5} = 1000 \]

Cutting speed: \[ \text{RPM} = \frac{200}{0.5} = 400 \]

Load Calculations:

Reflected torque:

\[ \text{Torque} = \frac{\text{Torque at screw}}{\text{GR (reducer ratio) } \times \eta_{GR} \text{ (reducer efficiency)}} \]

Using a practical ratio of 3 (see Note):

\[ \text{Torque} = \frac{19.3}{3 \times 0.9} = 7.1 \text{ lb } \cdot \text{ft} \]

Reflected speed:

\[ \text{Speed} = \text{Speed at screw } \times \text{GR(reducer ratio)} \]

Using a ratio of 3:

\[ N_{\text{Max}} = 1000 \times 3 = 3000 \text{ RPM} \]
\[ N_{\text{Cut}} = 400 \times 3 = 1200 \text{ RPM} \]

Note: Selecting an optimum reducer ratio is a function of many factors including maximizing torque multiplication, and reducing reflected load inertia while staying within practical limits of belt ratios or gearhead input speeds. A typical 3:1 reducer and coupling inertia for the size of motor needed has an inertia of 0.000251 lb-ft-s².

Component Selection:

Select system: B-404-C/Sx20

Performance Summary

\[ T_c = 9.7 \text{ lb-ft} \]
\[ T_p = 17.8 \text{ lb-ft} \] (17.5 lb-ft @ 3000 RPM)
\[ N_{\text{max}} = 5000 \text{ RPM} \]
\[ J_m = 0.000484 \text{ lb-ft-s}^2 \]

Acceleration Calculation:

Maximum acceleration is now calculated based on the inertia of all system components.

\[ \alpha = \frac{\text{Peak torque available}}{\text{System inertia}} \frac{\text{rad/s}^2}{\text{rad/s}^2} \]

\[ \alpha = \frac{T_p - T_f}{J_{\text{Slide}} + J_{\text{Screw}} \times \text{GR}^2 + J_{\text{Motor}} + \eta_{GR}} \]

Units: \[ T_p = \text{Peak torque of system in lb-ft} \]
\[ T_f = \text{Friction torque from previous calculation} \]
\[ J_{\text{Slide}}, J_{\text{Screw}} \text{ & } J_{\text{Motor}} = \text{Inertia of slide, screw reducer & motor in lb-ft-s}^2 \]
\[ \text{GR} = \text{Reducer ratio} \]
\[ \eta_{GR} = \text{Efficiency of reducer in } \%/100 \]

\[ \alpha = \frac{17.5 - 0.9}{0.0019 + 0.0016 + 0.000484 + 0.000251} = 14,223 \]

\[ \alpha = 14,223 \text{ rad/sec}^2 \times \left( \frac{0.5 \text{ in/rev}}{2\pi \text{ rad/rev} (3)} \right) = 377 \text{ in/s}^2 \]

Conclusion:

The B-404-C/Sx20 system provides adequate continuous and peak torque and maximum speed capability. The maximum acceleration requirement is easily satisfied and the encoder feedback requirement is met by setting the “encoder equivalent” output signal on the SERVOSTAR® Sx20 amplifier to 1024 lines/rev (4096 counts/rev or higher).
Application Example: Slide Drive continued

System Performance Specifications

<table>
<thead>
<tr>
<th>Performance Specification</th>
<th>Symbol</th>
<th>Units</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Cont. Torque @ Stall</td>
<td>$T_C$</td>
<td>N-m</td>
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<td>Motor Inertia</td>
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<tr>
<td></td>
<td></td>
<td>lb</td>
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</tr>
</tbody>
</table>

![Graph showing speed and torque characteristics of B-404-C/Sx20 (230 V)](image_url)
Application Example: Document Handling Machine

General Information:
This system continuously conveys 9 inch letters while maintaining a uniform gap between them. This requires rapid acceleration and deceleration of the conveyor.

Customer Specifications:
- Feeder (conveyor) inertia = 0.0090 oz-in-s²
- Drive wheel diameter = 60 mm
- Conveyor belt thickness = 7 mm
- Letter length = 9 inches (average)
- Static friction (TF) = 80 oz-in
- Gap distance for accel or decel = 20 mm
- Max line speed = 200 in/s

Speed Calculations:
\[ N_{\text{max}} = \frac{200 \text{ in/s} \times 60 \text{ s/min} \times 25.4 \text{ mm/in}}{2\pi \left( \frac{60}{2} + 7 \right) \text{ mm/rev}} \]
\[ N_{\text{max}} = 1131 \text{ RPM} \]

Time Calculations:
Time to accelerate or decelerate \((t_a, t_d)\)

For linear acceleration:
\[ d = \frac{1}{2} V_{\text{max}} t \]
\[ t_a, t_d = \frac{2 \text{ Gap}}{V_{\text{max}}} \]
\[ t_a = \frac{2(20 \text{ mm})}{200 \text{ in/s} \times 25.4 \text{ mm/in}} \]
\[ t_a = 0.00787 \text{ s} \]

The letter moves 20mm (0.787 inches) during accel and decel periods. The remaining move time for the move will be at the running speed.

\[ t_r = \frac{9 \text{ in} - 2(0.787 \text{ in})}{200 \text{ in/s}} \]
\[ t_r = 0.0371 \text{ s} \]

Torque and Inertia Calculations:
RMS Torque is calculated as follows:
\[ T_{\text{RMS}} = \sqrt{\frac{T_1^2 t_1 + T_2^2 t_2 + T_3^2 t_3}{t_1 + t_2 + t_3}} \]
Application Example: Document Handling Machine continued

Where:

\[ T_1 = T_a (\text{Accel Torque}) + T_F (\text{Friction Torque}) \]

\[ T_1 = \frac{(\text{Total Inertia, oz-in-s}^2) \times N (\text{RPM})}{9.55 \times t_a} \]

\[ J_T = J_L (\text{Load Inertia}) + J_M (\text{Motor Inertia}) \]

\[ J_L = J_{\text{Feeder}} + J_{\text{Letter}} (\text{mrad}^2) \]

\[ J_L = 0.0090 \text{ oz-in-s}^2 + \frac{2.0 \text{ oz}}{386.1 \text{in/s}^2} \left(30 + 7 \text{ mm} \right)^2 \]

\[ J_L = 0.01999 \text{ oz-in-s}^2 \]

(An initial motor sizing estimation is necessary so that motor inertia can be included in acceleration calculations.)

\[ J_M = 0.0157 \text{ oz-in-s}^2 \]

(MT306A1 Kollmorgen GOLDLINE® XT Motor)

\[ J_T = J_L = 0.01999 + 0.0157 \text{ oz-in-s}^2 \]

\[ J_T = 0.03569 \text{ oz-in-s}^2 \]

\[ T_a = \frac{0.03569 \text{ oz-in-s}^2 (1311 \text{RPM})}{9.55 (0.00787 \text{ s})} \]

\[ T_a = 623 \text{ oz-in} \]

\[ T_1 = 623 + 80 \text{ oz-in} \]

\[ T_1 = 703 \text{ oz-in} \]

\[ t_1 = t_a = 0.00787 \text{ s} \]

\[ T_2 = T_F (\text{static friction}) = 80 \text{ oz-in} \]

\[ t_2 = t_r (\text{running time}) = 0.0371 \text{ s} \]

\[ T_3 = T_d (\text{decel torque}) - T_1 (\text{friction torque}) \]

\[ T_d = \frac{J_T (\text{oz-in-s}^2) \times N (\text{RPM})}{9.55 \times t_d} \]

\[ t_d (\text{decel time}) = t_a = 0.00787 \text{ s} \]

\[ T_{\text{RMS}} = \sqrt{\left(\frac{703^2}{2(0.00787)^2} + \frac{0.0371^2}{2(0.00787)^2} + \frac{0.0157^2}{2(0.00787)^2} + \frac{0.03569^2}{2(0.00787)^2}\right) \left(80^2 + (80)^2 + (-543)^2 + 0.03569^2 \right) oz \cdot \text{in}} \]

\[ T_{\text{RMS}} = 349 \text{ oz-in} \]

Conclusion:

Observation of the system MT306A1/Cx03 (230V) speed / torque curve shows the acceptability of this system to meet maximum speed, peak, and RMS torque requirements.
**System Performance Specifications**

<table>
<thead>
<tr>
<th>Performance Specification</th>
<th>Symbol</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cont. Torque @ Stall</td>
<td>$T_C$</td>
<td>N-m/oz-in</td>
</tr>
<tr>
<td>Peak Torque @ Stall</td>
<td>$T_P$</td>
<td>N-m/oz-in</td>
</tr>
<tr>
<td>Cont. Power</td>
<td>$W_{rated}$</td>
<td>Watts/hp</td>
</tr>
<tr>
<td>Max Speed</td>
<td>$N$</td>
<td>RPM</td>
</tr>
<tr>
<td>Motor Inertia</td>
<td>$J_m$</td>
<td>kg-cm$^2$/oz-in-s$^2$</td>
</tr>
<tr>
<td>Motor Weight</td>
<td>$Wt$</td>
<td>kg/lb</td>
</tr>
</tbody>
</table>

For MT306A1/ Cx03 (230 V):

- Cont. Torque @ Stall: $T_C = 3.3$ N-m, 467.3 oz-in
- Peak Torque @ Stall: $T_P = 8.6$ N-m, 1218 oz-in
- Cont. Power: $W_{rated} = 820$ Watts; $hp_{rated} = 1.1$ hp
- Max Speed: $N = 2500$ RPM
- Motor Inertia: $J_m = 1.1$ kg-cm$^2$/oz-in-s$^2$; 0.0157
- Motor Weight: $Wt = 3.8$ kg; 8.4 lb
### TORQUE CONVERSION TABLE

<table>
<thead>
<tr>
<th>A</th>
<th>dyne-cm</th>
<th>gm-cm</th>
<th>oz-in</th>
<th>kg-cm</th>
<th>lb-in</th>
<th>Newton-m</th>
<th>lb-ft</th>
<th>kg-m</th>
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<tbody>
<tr>
<td>dyne-cm</td>
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<td>1.01972x10^3</td>
<td>1.41612x10^5</td>
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<td>oz-in</td>
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<td>kg-cm</td>
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To convert from A to B, multiply by entry in table.

### ROTARY INERTIA CONVERSION TABLE

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<tr>
<th>A</th>
<th>gm-cm²</th>
<th>oz-in²</th>
<th>gm-cm² or n-m-sec²</th>
<th>kg-cm²</th>
<th>kg-m² or n-m-sec²</th>
<th>lb-in²</th>
<th>oz-in·sec²</th>
<th>lb-ft²</th>
<th>kg-cm·sec²</th>
<th>lb-in·sec²</th>
<th>lb-ft·sec² or slug-ft²</th>
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</thead>
<tbody>
<tr>
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To convert from A to B, multiply by entry in table.