White Paper

Evolving the Rules

of Inertia Matching

KOLLMORGEN

The accepted principle of matching motor to load inertia is no longer pertinent with today's faster processors and advanced control algorithms.

This outdated method increases costs and adds unnecessary mass in applications where load inertia is high and the continuous torque requirements are low. **Motor inertia is only one consideration when developing an optimally performing solution requiring good bandwidth and servo stiffness.**

RULE ORIGINATION

Inertia matching was believed to address the stable control of a driven load connected to a servo motor. During the 70's, when brush type servo motors began to replace hydraulics in the machine tool world, designers calculated the load inertia, torque, and speed requirements based on the expected performance of the machine. When selecting a motor to meet the needed torque and speed requirements, if the motor to load inertia was not close to a 1:1 match, substituting a motor with higher inertia or using a gearbox (which would reduce the reflected inertia seen by the servo motor) would be considered, thus increasing the cost of the system. Though optimal power transfer does occur when inertias are matched, that does not guarantee an efficiently operating system. Ideally, total system inertia should be reduced to expend less energy. A larger motor, however, increases the torque requirements to accelerate the added motor inertia.

There are more considerations to application sizing than just inertia matching. During the transition of hydraulic to electric motors, quick analysis of complete mechanical and control systems was limited by available technology. The construction of these closed loop servo systems includes elements that can dramatically affect machine performance, such as the motor, attached feedback device, coupling to the load, and the capabilities to tune the servo loops. To provide good performance, servo loops are tuned to operate with the desired bandwidth and servo stiffness, which optimizes the response to controller commands with minimal overshoot. The servo motor is controlled by a servo drive utilizing current, velocity, and position loops. Each loop is tuned to create an enhanced system response through stability, quick reactions to torque or velocity disruptions, and smooth operation. In the early years, tuning servo loops used discrete components and potentiometers to adjust loop gains determined by experimentation. Limited analytical tools and processing power combined with discrete components dictated a close inertia match between motor and load. Even as processors and analytics improved and digitally tuned servo loops were developed, the old rule of an optimal 1:1 match continued to perpetuate.

TECHNOLOGY ADVANCEMENT

With the advent of brushless motor technology, high energy NeFeB magnets, and digital tuning loops, the inertia matching protocol met with new complications. High energy magnets located on the rotor significantly reduced motor inertia in comparison to its brush type predecessors. Motors meeting the application's required continuous and peak torque capabilities had higher load to motor inertia mismatches.

While servo motor digital tuning loops made it considerably easier to adjust gains and filters to provide stable control; low processor speeds, low resolution feedback devices, and other limiting factors led to the development of brushless motor options with added inertia.

Increased processing power allowed the complex analytics to create accurate mathematical modeling and simulation of system responses. Modern capabilities incorporating powerful integrated servo drive tools create interactive analytics of complex mechanical systems simplifying the optimization of servo systems. Advanced analytics also allow the machine designer to understand in detail the precise fingerprint of the mechanical system and how to address performance limitations.

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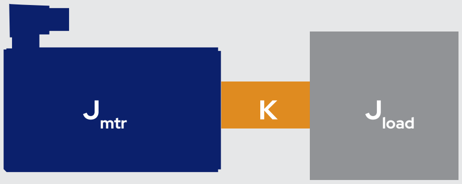
COMPLIANCE – THE BANE OF HIGH BANDWIDTH SOLUTIONS

Compliance in a mechanical system is the natural springiness of the mechanisms between the driven load and the motor that creates delayed response times leading to reduced system bandwidth. Introduce a large inertia mismatch into the system and the problem is magnified, as in the case of a small motor with enough torque to move an exceptionally large load but connected via a coupling device. When the small motor quickly applies torque to the large load, the larger load will hesitate to respond, since an object at rest wants to stay at rest. The delay is a result of coupling compliance between the motor and load that introduces windup before the load begins to move. As the load eventually syncs up with the motor, the large inertia will overshoot the target speed, causing the smaller motor to adjust by slowing down. When the system adjusts the overspeed of the large inertia, the target speed is again passed, triggering the small motor to adjust once more. This continued cycling creates resonance and an unstable system.

Most mechanical systems can be mathematically modeled and simulated using various excitation frequencies to quickly identify the frequency response - where a resonance occurs. The bandwidth of a system can never exceed the initial anti-resonance point of the system. The goal of increasing bandwidth is to push the initial resonance frequency higher by identifying and addressing the cause of the resonance. In a compliant system, as the compliance or springiness increases, the frequency of the initial resonance point reduces which decreases bandwidth. When the driven load is directly coupled to the motor to minimize compliance, the mismatch is mitigated – increasing the initial resonance frequency and creating a higher bandwidth system.



$$J_e = \frac{J_{mtr}\,J_{load}}{J_{mtr} + J_{load}} \qquad F_{antires} = \sqrt{\frac{K}{J_{load}}} \qquad F_{res} = \sqrt{\frac{K}{J_e}}$$

As the ratio between J_{load} and J_{mtr} increases J_e will approach J_{mtr} so if J_{mtr} decreases, then J_e decreases causing the resonant frequency to go up. Increasing K also causes frequency to go up. The anti-resonant frequency will not change since load inertia is constant but will increase in stiffness. Note that frequency (F) is in rad/sec for these equations.

INCREASED STIFFNESS AND REDUCED SYSTEM INERTIA

Mathematical models representing a mechanical system show that the ultimate solution for a higher bandwidth and cost-effective system is to increase the mechanical stiffness and to reduce total system inertia.

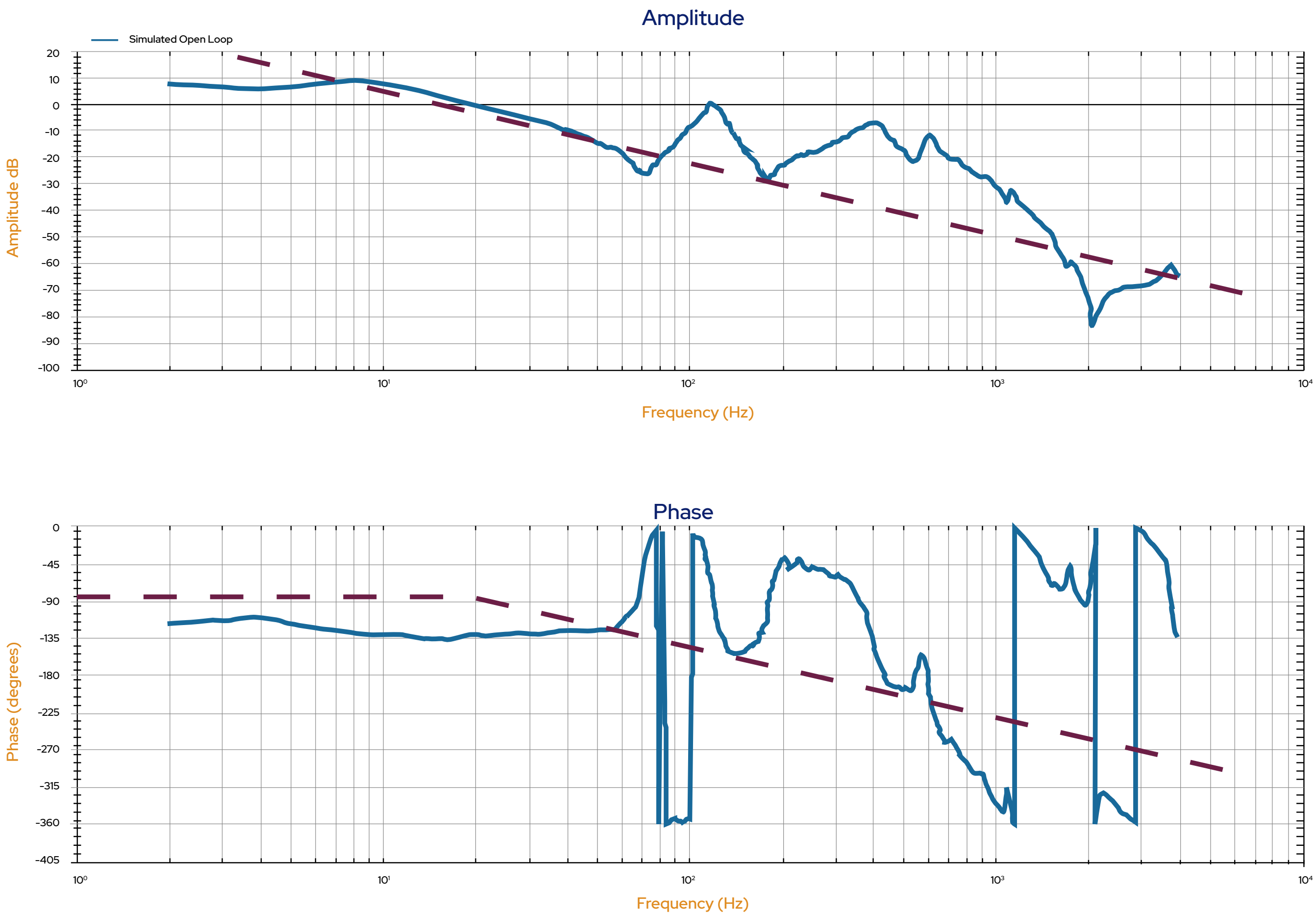
Consider a direct drive solution where the load is directly coupled to the motor with near zero compliance. Precisely controlling the system with good bandwidth can be achieved even with inertia mismatches exceeding 1000:1. In an extremely stiff (non-compliant) system, the servo system should be sized to provide the necessary torque to move the system inertia in the manner required by the specific application. Since direct drive solutions are not suited to all applications, compliant elements will be introduced into the system. Present advanced analytical tools readily identify the compliant elements that reduce system performance.

THE BODE PLOT

The bode plot is a powerful analytical tool consisting of two charts illustrating the frequency response of an injected signal to identify the amplitude and phase lag of the system. Bandwidth, phase and gain margins, resonance and anti-resonance points are just a few elements captured on a bode plot. It also provides clues to inertia mismatch, number of connected bodies, friction levels, and identifies the open and closed loop bandwidth, phase and gain margins, and resonance frequencies. This information is invaluable in tuning the system for optimal performance by adjusting loop gains, installing various digital filtering, and considering adjustments to the mechanics.

PLOT MEASUREMENTS

Bode plots consist of a gain and phase plot and will have characteristics as shown below.

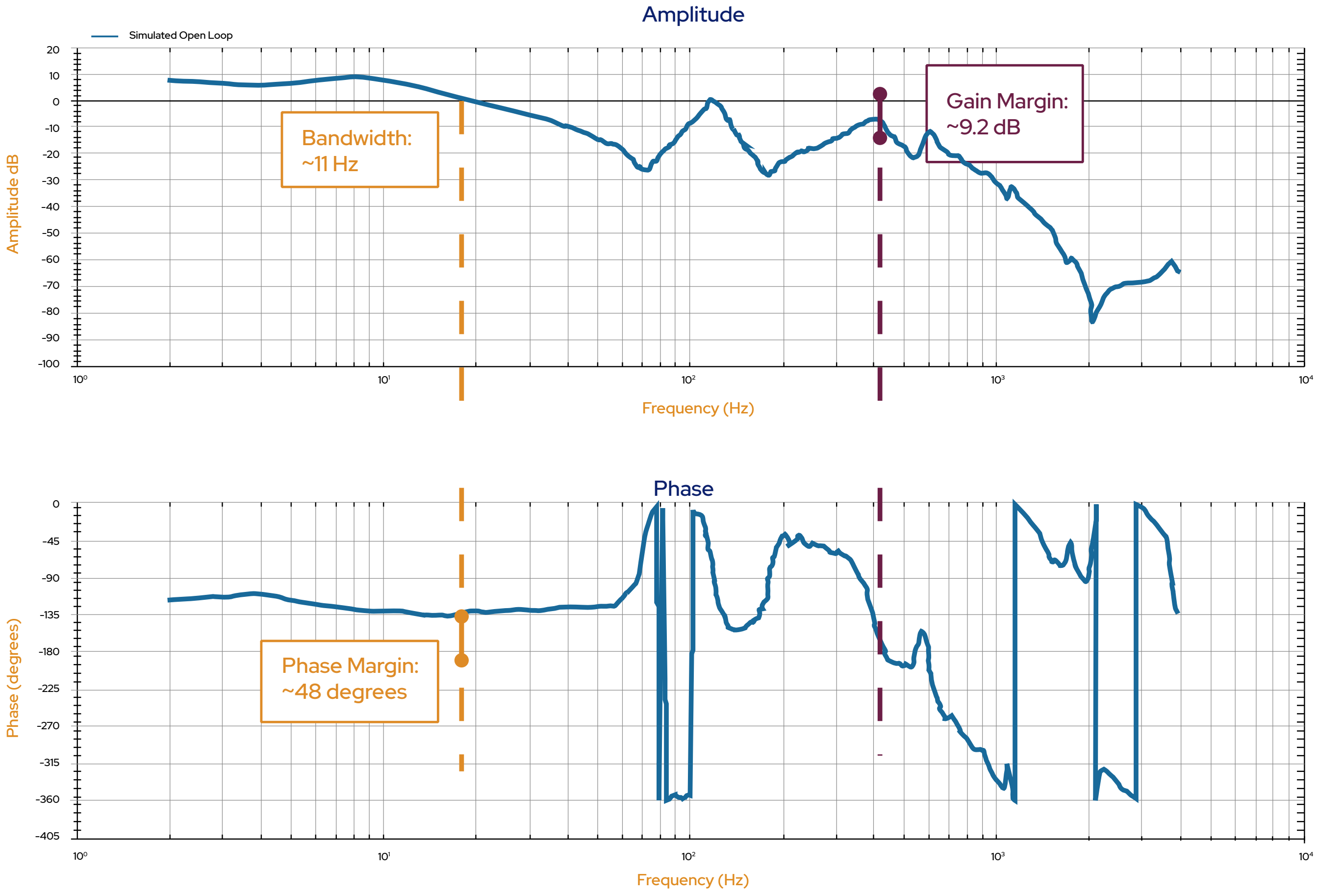


In a perfect system, we expect the Amplitude plot to be a straight negative slope, -20dB/decade. The Phase plot should start at -90° and drop at a negative slope from the point the amplitude crosses zero dB.

CALCULATING BANDWIDTH - PHASE AND GAIN MARGINS

Using a valid bode plot, the open and closed loop bandwidths can be determined, as well as the associated gain and phase margins. The bandwidth is represented by the frequency at which the open loop plot reaches 0 dB (~11 Hz). The phase margin is the number of degrees above -180 degrees (~48 degrees) and the gain margin is the amplitude measurement corresponding to a phase of -180 degrees (~9.2 dB).

The following example illustrates how to successfully optimize both performance and cost by applying improved system stiffness to the solution, without concern for inertia mismatch.



A 3-axis laser cutting machine was designed using the inertia matching approach for selecting the axis motors. A redesign was desired to reduce cost and improve performance of the machine. A review of the application requirements showed that alternate motor solutions could increase the system resonance point to allow additional gain and phase margins and improved stability. The selected servo motor reduced the total system inertia, increased the stiffness of the axis with a larger shaft diameter (higher resonant frequency), and provided higher power density in a smaller package. The increased shaft stiffness reduced compliance which improved performance.

The following chart illustrates the improved performance and cost savings by the elimination of the inertia matching approach in favor of increased mechanical stiffness and reduced inertia.

AXIS	Original Jm (kg-cm^2)	New Jm (kg-cm^2)	Load inertia (kg-cm^2)	Original Inertia mismatch	New inertia mismatch	% Increase	% cost savings
X	120	67.7	256.75	2.14	3.79	77%	17%
Y	17	4.58	9.56	0.56	2.09	273%	34%
Z	121.6	80	29.4	0.24	0.37	54%	17%

CONCLUSION

Modern servo drives with advanced tuning capabilities and high-performance servo motor designs that incorporate high resolution feedback eliminate load to motor inertia mismatch concerns. Proper application sizing and best practices in designing a stiff mechanism lead to a high-performance motion system capable of higher bandwidths, improved move and settle times, and robust dynamic control.

Want to Learn More?

Additional related topics include mechatronics, control theory, digital filtering techniques, servo loop tuning, mathematical modeling, and mechanical resonances. Kollmorgen offers a multi-day, advanced servo tuning course that includes the hands-on application of bode plots and tuning techniques using digital filtering. Similar topics are covered in these publications:

White Papers (click to read)

Sizing and Selecting Servos

Energy Management of a Servo Motor – Effects of Inertia Ratio

Integrated Model Based Machine Design

Simplified Machine Design Approach for Optimal Servo Motor Control

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